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New Prediction For Leptonic θ_{13}

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Abstract

An extension of the neutrino sector with two right handed singlet neutrinos responsible for Dirac neutrino masses is discussed. We show that this setup with flavor symmetry can give large solar and atmospheric neutrino mixings and suppressed θ_{13}. The flavor symmetry $\mathcal{U}(1) \times S_4$ is shown to lead to $\theta_{23} \simeq \pi/4$ and a new predictive formula for the $\theta_{13}(\simeq 0.015)$.

Recent atmospheric [1] and solar [2] neutrino data have confirmed the neutrino oscillations. A global analysis [3] for the oscillation parameters yields

$$|\Delta m^2_{\rm atm}| = 2.4 \cdot \left(1^{+0.21}_{-0.26}\right) \times 10^{-3}~{\rm eV}^2~, \quad \sin^2\theta_{23} = 0.44 \cdot \left(1^{+0.41}_{-0.22}\right)~,$$
$$\Delta m^2_{\rm sol} = 7.92 \cdot (1 \pm 0.09) \times 10^{-5}~{\rm eV}^2~, \quad \sin^2\theta_{12} = 0.314 \cdot \left(1^{+0.18}_{-0.15}\right)~. \tag{1}$$

The third leptonic mixing angle θ_{13} has not yet been measured but a useful upper bound exists,

$$\theta_{13} \lesssim 0.2~, \tag{2}$$

provided by the CHOOZ experiment [4]. The corresponding mass scales in (1) indicate new physics beyond the SM or beyond its minimal SUSY extension - MSSM. Looking at (1) and (2), together with neutrino mass generation, one can try to understand the origin of two large mixing angles ($\theta_{12} \simeq 35^{\rm o}$, $\theta_{23} \simeq 42^{\rm o}$) and a suppressed third angle ($\theta_{13} \lesssim 12^{\rm o}$). Non-zero value of θ_{13} would cause the CP violation in the neutrino oscillations. In addition to the values of θ_{13} and the phase δ, the sign of $\Delta m^2_{\rm atm}$ is also unknown. This sign is directly related to whether neutrinos have normal or inverted hierarchical mass pattern. Planned experiments are expected to shed more light to these important issues. These will give new selection rules for theoretical model building and will rule out many existing scenarios. On the other hand, it still remains a great challenge to build self consistent scenarios which give natural explanation of bi-large neutrino mixings and predict the value of θ_{13}. In this respect, the symmetry principle seems to be the most powerful tool and we will pursue this approach here. Numerous attempts have been made [5]- [10] to explain and/or predict the suppressed value of θ_{13} within different setups. In this paper we suggest new framework which naturally gives bi-large neutrino mixings and suppressed θ_{13}. Our proposal works for both non SUSY and SUSY scenarios. Since supersymmetry has strong theoretical and phenomenological motivations, we stick here with the SUSY description. We consider extension of the MSSM by two right handed neutrino (RHN) superfields $N_{1,2}$ and flavor symmetry G_f (for earlier work with similar extension see [11]). An extension of the SM (plus a Z_2 symmetry) with RHNs and an additional Higgs doublet with a tiny VEV has recently been considered [12] for understanding the tiny neutrino masses as an alternative to the usual sea-saw mechanism. The neutrinos are Dirac particle, and the model of [12] has interesting phenomenological implications, specially for the Higgs boson searches. In this work, our focus is different and thus we restrict ourself without additional Higgs doublets. Neutrino mass suppression will happen due to tiny Yukawa couplings($\lesssim 10^{-11}$) guaranteed by flavor symmetry. We show that with $G_f = \mathcal{U}(1)$ the value of θ_{13} is estimated to be $\sim \frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}} \approx 0.03$, while with $G_f = \mathcal{U}(1) \times S_4$, an accurate prediction for θ_{13} is possible $\theta_{13} \simeq 0.014$.

- **Case with $\mathcal{U}(1)$ flavor symmetry**

Let us start the discussion with neutrino sector augmented by two RHNs N_1, N_2 and $G_f = \mathcal{U}(1)$ flavor symmetry. For $\mathcal{U}(1)$ symmetry breaking we introduce MSSM singlet superfield X charged under $\mathcal{U}(1)$ which has VEV in its scalar component

$$\frac{\langle X \rangle}{M_{\rm Pl}} \equiv \epsilon~. \tag{3}$$

This is realized naturally if $\mathcal{U}(1)$ is an anomalous gauge symmetry often emerging from superstrings [13]. Then, X's VEV can be fixed by cancelation condition of the Fayet-Iliopoulos D_A term: $\xi + Q_X |X|^2 = 0$.

Consider the following $\mathcal{U}(1)$ charge assignment:

$$Q_X = -1 \ , \quad Q_{h_u} = 0 \ , \quad Q_{l_i} = q \ , \quad Q_{N_1} = n + 1 - q \ , \quad Q_{N_2} = n - q \ , \tag{4}$$

where n is a positive integer and l_i denote $SU(2)_L$ lepton doublet superfields, while h_u is up type MSSM Higgs superfield. The relevant couplings allowed by $\mathcal{U}(1)$ symmetry and leading to the Dirac neutrino masses are

$$M_\nu := \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \!\!\begin{array}{c} \begin{array}{cc} N_1 & N_2 \end{array} \\ \left(\begin{array}{cc} a\epsilon & 0 \\ \epsilon & \alpha \\ b\epsilon e^{\mathrm{i}\delta} & 1 \end{array} \right) \end{array} \epsilon^n h_u \ , \tag{5}$$

where instead of powers of $X/M_{\rm Pl}$ we have substituted ϵ according to the notation of Eq. (3) and a, b, α are dimensionless constants of the order one. We have set (1,2) entry of the matrix in (5) to zero. This can be achieved by proper rotation of $N_{1,2}$ states without any loss of generality, because this basis redefinition does not change hierarchical structure between remaining matrix elements. Moreover, by proper phase redefinitions only one complex phase δ remains in the (3,1) entry.

The mass eigenvalues and the unitary matrix transforming the left handed neutrinos upon diagonalization of (5) can be easily found from the diagonalization of the matrix $M_\nu M_\nu^\dagger$. For convenience we will write it in a form:

$$M_\nu M_\nu^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha \\ 0 & \alpha & 1 \end{pmatrix} \tfrac{m^2}{1+\alpha^2} + \begin{pmatrix} a^2 & a & abe^{-\mathrm{i}\delta} \\ a & 1 & be^{-\mathrm{i}\delta} \\ abe^{\mathrm{i}\delta} & be^{\mathrm{i}\delta} & b^2 \end{pmatrix} \tfrac{m^2\epsilon^2}{1+\alpha^2} \ ,$$

$$\text{with} \qquad m^2 = (1+\alpha^2)\epsilon^{2n} \langle h_u^{(0)} \rangle^2 \ . \tag{6}$$

We will take $\epsilon \ll 1$ which is natural from the symmetry viewpoint and leads to the neutrino mass pattern with a normal hierarchy. In (6) we have split $M_\nu M_\nu^\dagger$ in two parts. First one including 2×2 matrix block of rank one and second one is 3×3 rank one matrix. The first leading part is responsible for the mass m_3 the heaviest neutrino and for θ_{23} mixing. The second, sub-leading part, gives m_2 and θ_{12}. In particular, we have

$$m_1 = 0 \ , \quad m_2 \simeq \sqrt{\Delta m^2_{\rm sol}} \simeq \frac{m\epsilon}{1+\alpha^2} \left[a^2(1+\alpha^2) + |1 - b\alpha e^{-\mathrm{i}\delta}|^2 \right]^{1/2} \ , \quad m_3 \simeq \sqrt{\Delta m^2_{\rm atm}} \simeq m \ . \tag{7}$$

Furthermore, for diagonalizing unitary matrices we have

$$U_\nu^T M_\nu M_\nu^\dagger U_\nu^* = (M_\nu^{\rm diag})^2 \ , \qquad \text{with} \qquad U_\nu = U_{23} U_{13} U_{12} \ , \tag{8}$$

where U_ν is the lepton mixing matrix (assuming that the charged lepton mass matrix is diagonal) and

$$U_{23} = \begin{pmatrix} 1 \ , & 0 \ , & 0 \\ 0 \ , & c_{23} \ , & s_{23} \\ 0 \ , & -s_{23} \ , & c_{23} \end{pmatrix} , \quad c_{23} \equiv \cos\theta_{23} \ , s_{23} \equiv \sin\theta_{23} \ , \quad \tan\theta_{23} = \alpha \ , \tag{9}$$

$$U_{12} \simeq \begin{pmatrix} c_{12}e^{\mathrm{i}\omega} \ , & s_{12}e^{\mathrm{i}\omega} \ , & 0 \\ -s_{12} \ , & c_{12} \ , & 0 \\ 0 \ , & 0 \ , & 1 \end{pmatrix} , \qquad U_{13} \simeq \begin{pmatrix} c_{13}e^{\mathrm{i}\phi} \ , & 0 \ , & s_{13}e^{\mathrm{i}\phi} \\ 0 \ , & 1 \ , & 0 \\ -s_{13} \ , & 0 \ , & c_{13} \end{pmatrix} ,$$

$$\phi = -\mathrm{Arg}\left(s_{23} + bc_{23}e^{\mathrm{i}\delta}\right) \ , \quad \omega = -\phi - \mathrm{Arg}\left(c_{23} - bs_{23}e^{-\mathrm{i}\delta}\right) \ ,$$

$$\tan\theta_{12} \simeq \frac{a}{|c_{23} - bs_{23}e^{-\mathrm{i}\delta}|} \ , \qquad \tan\theta_{13} \ \simeq \frac{a\epsilon^2}{1+\alpha^2}|s_{23} + bc_{23}e^{\mathrm{i}\delta}| \ . \tag{10}$$

We see that for $a \sim b \sim \alpha \sim 1$ we have naturally bi-large mixing $\tan\theta_{23} \sim 1$ and $\tan\theta_{12} \sim 1$. Moreover, using (7)-(10), we can express the third mixing angle θ_{13} as

$$\tan\theta_{13} \simeq \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}} \frac{\tan\theta_{12}\tan\theta_{23}}{1+\tan^2\theta_{12}} \left| \frac{1 + b\cot\theta_{23}e^{\mathrm{i}\delta}}{1 - b\tan\theta_{23}e^{-\mathrm{i}\delta}} \right| \ . \tag{11}$$

We see that θ_{13} is suppressed by factor $\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}$ ($\ll 1$), which appears due to the specific texture of Eq. (6). The texture similar to (6) for active Majorana neutrinos was considered in [7], while in the scenarios of refs. [8], [9] was derived by symmetries. Here, however, the neutrinos are purely Dirac type and the texture (6) is the 'squire' ($M_\nu M_\nu^\dagger$) of Dirac mass matrix M_ν. This cause stronger suppression of the θ_{13} angle (note however that the model of [8] deals with Majorana neutrinos and due to S_3 flavor symmetry θ_{13} is still suppressed by $\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}$ factor).

Although the unknown parameter b enters in (11), from the naturalness viewpoint we expect that $b \sim 1/3{-}3$ and therefore suppression always will happen giving $\theta_{13} \sim 10^{-2}$. The θ_{13} might enhanced when $|1 - b\tan\theta_{23}e^{-\mathrm{i}\delta}| \equiv \tilde{b} \ll 1$. However, this can not be realized because the same combination sets the values of $\tan\theta_{12}$ and $\sqrt{\Delta m^2_{\mathrm{sol}}}$ [see Eqs . (10) and (7)]. The flavor symmetry $\mathcal{U}(1)$ used here, besides the suppression of θ_{13}, also gives an explanation of small value of $\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}} \sim \epsilon^2$, giving $\epsilon \sim 1/6 - 1/5$. Also, $\sqrt{\Delta m^2_{\mathrm{atm}}} \simeq \epsilon^n \langle h_u^{(0)} \rangle \simeq 0.05$ eV requires $n = 16, 17$. $\mathcal{U}(1)$ flavor symmetry does not give hint about values of b, δ and therefore does not allow for θ_{13}'s accurate prediction. In addition, although θ_{23} and θ_{12} are naturally large it is desirable to have more clue (from the theory) about their values. Below we present a more concrete model which gives strict predictions for θ_{13} and θ_{23}.

- **Case with $G_f = \mathcal{U}(1) \times S_4$ and prediction of θ_{13}.**

The expression (11) obtained above is very intriguing since it gives naturally suppressed θ_{13}. However, if by some reason $b = 0$ then also δ becomes irrelevant and we will be able to express θ_{13} in terms of oscillation parameters which are already measured! We now show that this can be achieved if together with $\mathcal{U}(1)$ we introduce non-Abelian discrete symmetry. The latter have been proven to be powerful in obtaining various predictive relations [14], [16], [10]. We will treat three left handed lepton doublets l_i as a triplet in a family space. For this purpose either S_4 or A_4 can be used. For demonstration, we discuss an example with $G_f = \mathcal{U}(1) \times S_4$. Thus $\vec{l} \equiv (l_1, l_2, l_3) \sim \mathbf{3_1}$ and $N_{1,2}$ and h_u are S_4 singlets $\mathbf{1_1}$ [15]. For S_4 breaking and desirable neutrino mass matrix generation we introduce three scalar superfields $\vec{S}_1, \vec{S}_2, \vec{A} \sim \mathbf{3_1}$. We use the $\mathcal{U}(1)$ charge assignment for $X, h_u, \vec{l}, N_1$ as given in (4) and take $Q_{N_2} = n - q - q_A$, $Q_{\vec{A}} = q_A$, $Q_{\vec{S}_1} = Q_{\vec{S}_2} = 0$. Then couplings allowed by

introduced symmetries are

$$\frac{\epsilon^{n+1}}{M_{\rm Pl}}\vec{l}\left(\vec{S}_1+\vec{S}_2\right)N_1 h_u+\frac{\epsilon^n}{M_{\rm Pl}}\vec{l}\vec{A}N_2 h_u\ . \tag{12}$$

For S_3 triplet scalars we will consider the following VEV configuration

$$\langle\vec{S}_1\rangle=(V_1\ ,\ 0\ ,\ 0)\ ,\quad \langle\vec{S}_2\rangle=(0\ ,\ V_2\ ,\ 0)\ ,\quad \langle\vec{A}\rangle=(0\ ,\ V\ ,\ {\rm i}V)\ . \tag{13}$$

This VEV structure can be obtained from a simple superpotential. For example, by introducing the singlets superfields X_1, X_2, X_A with $\mathcal{U}(1)$ charges $Q_{X_1}=Q_{X_2}=0, Q_{X_A}=-2q_A$ respectively, allowed superpotential couplings are[1]

$$W(\vec{S}_{1,2},\vec{A})=X_1\left(\vec{S}_1^2-V_1^2\right)+X_2\left(\vec{S}_2^2-V_2^2\right)+X_A\vec{A}^2\ . \tag{14}$$

Imposing F-flatness conditions one can easily see that solutions in (13) are obtained with $\langle X_{1,2}\rangle=\langle X_A\rangle=0$. Substituting (13) in (12) we obtain

$$M_\nu=\begin{array}{c} \\ l_1\\ l_2\\ l_3\end{array}\!\!\begin{array}{c}\begin{array}{cc} N_1 & N_2\end{array}\\ \begin{pmatrix} V_1\epsilon & 0\\ V_2\epsilon & V\\ 0 & {\rm i}V\end{pmatrix}\end{array}\frac{\epsilon^n}{M_{\rm Pl}}h_u\ , \tag{15}$$

Because of the absence of (1,2) and (3,1) entries all complex phases can be rotated away from the mass matrix (15). Comparing now Eq. (15) with (5) we have $\alpha=1$ and $b=0$. Therefore, using (9) and (11) we get two predictive relations

$$\tan\theta_{23}\simeq 1\ , \tag{16}$$

and

$$\tan\theta_{13}\simeq\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}\frac{\tan\theta_{12}}{1+\tan^2\theta_{12}}\ . \tag{17}$$

Therefore, the atmospheric mixing is maximal ($\theta_{23}\simeq\pi/4$) which is favored by the neutrino data. For the third mixing angle we obtain

$$\theta_{13}=0.015\cdot\left(1^{+0.53}_{-0.28}\right)\ . \tag{18}$$

Uncertainty in (18) is mostly due to uncertainties in the measured values of $\Delta m^2_{\rm atm}$ and $\sin^2\theta_{12}$ in Eq. (1). This prediction can be tested in the planned future neutrino experiments. Also, more accurate measurements of the solar and atmospheric mass difference squares, and the solar mixing angle will tighten the range of this prediction.

In conclusion, we have shown that extension of the neutrino sector with two RHN states and specifically selected flavor symmetries can provide naturally bi-large neutrino mixings and suppressed value of θ_{13}. The flavor symmetry $\mathcal{U}(1)\times S_4$ looks very promising for obtaining accurate predictions of the θ_{23} and θ_{13} angles. We note that it is important that the possible corrections from

[1] Similar interactions within different scenarios have been considered in refs. [16], [10].

the charged lepton sector maintain the predictions derived in the neutral lepton sector. This can be insured by specific breaking of the flavor symmetry in the charged sector (see for instance [10]). It would be interesting to see how the ideas suggested in this paper might work within various Grand Unified Theories.

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References

[1] S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **85**, 3999 (2000).

[2] S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **86** (2001) 5651; K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90** (2003) 021802; B. Aharmim *et al.* [SNO Collaboration], Phys. Rev. C **75** (2007) 045502.

[3] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **6** (2004) 122; G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, [hep-ph/0506307].

[4] M. Apollonio *et al.*, Eur. Phys. J. C **27** (2003) 331.

[5] K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552** (2003) 207; T. Kitabayashi and M. Yasue, Phys. Rev. D **67** (2003) 015006; Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **594** (2004) 177; A. S. Joshipura, hep-ph/0411154; P. F. Harrison and W. G. Scott, Phys. Lett. B **594** (2004) 324; N. Haba and K. Yoshioka, hep-ph/0511108; R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72** (2005) 053001; B. C. Chauhan, M. Picariello, J. Pulido and E. Torrente-Lujan, Eur. Phys. J. C **50** (2007) 573; See also references therein.

[6] K. Milton, S. Nandi and K. Tanaka, Phys. Rev. D **25** (1982) 800; A. Aranda, C. D. Carone and P. Meade, Phys. Rev. D **65** (2002) 013011; N. V. Cortez and M. D. Tonasse, Phys. Rev. D **72** (2005) 073005; Y. Cai and H. B. Yu, Phys. Rev. D **74** (2006) 115005; Y. H. Ahn, C. S. Kim, S. K. Kang and J. Lee, Phys. Rev. D **75** (2007) 013012; H. K. Dreiner, C. Luhn, H. Murayama and M. Thormeier, arXiv:0708.0989 [hep-ph]; For a review of the models see M. G. Albrow *et al.*, hep-ex/0509019; C. H. Albright, M. C. Chen, hep-ph/0608137.

[7] P. H. Frampton, S. L. Glashow and T. Yanagida, Phys. Lett. B **548**, 119 (2002); M. Raidal and A. Strumia, Phys. Lett. B **553** (2003) 72; A. Ibarra and G. G. Ross, Phys. Lett. B **575** (2003) 279.

[8] S. L. Chen, M. Frigerio and E. Ma, Phys. Rev. D **70** (2004) 073008.

[9] Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **633** (2006) 595; Nucl. Phys. B **772** (2007) 133.

[10] K. S. Babu, A. G. Bachri and Z. Tavartkiladze, arXiv:0705.4419 [hep-ph].

[11] S. F. King, Nucl. Phys. B **576** (2000) 85.

[12] S. Gabriel and S. Nandi, hep-ph/0610253.

[13] E. Witten, Nucl. Phys. B **188** (1981) 513; W. Fischler, H. P. Nilles, J. Polchinski, S. Raby and L. Susskind, Phys. Rev. Lett. **47** (1981) 757; M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B **289** (1987) 589.

[14] S. Pakvasa and H. Sugawara, Phys. Lett. B **73** (1978) 61; E. Ma and G. Rajasekaran, Phys. Rev. D **64** (2001) 113012; J. Kubo, A. Mondragon, M. Mondragon and E. Rodriguez-Jauregui, Prog. Theor. Phys. **109** (2003) 795; K. S. Babu and J. Kubo, Phys. Rev. D **71** (2005) 056006; F. Caravaglios and S. Morisi, hep-ph/0510321; I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B **644** (2007) 153; C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP **0606** (2006) 042; C. S. Lam, arXiv:0708.3665 [hep-ph]; and references therein.

[15] For the properties of S_4 group and other references see C. Hagedorn *et al.* in [14].

[16] R. Barbieri, L. J. Hall, G. L. Kane and G. G. Ross, hep-ph/9901228.